PreCalculus
Trig Identities Group Test v3
Page 1

Name Solution Key

Score___________

For Problem 1-6, use:

(15, -8) is on the terminal side of A and $270° \le A \le 360°$

$\sin A = \frac{-8}{17}$ $\cos A = \frac{15}{17}$ $\tan A = \frac{-8}{15}$

$\cos B = 3/5$ and $-90° \le B \le 0°$; and

$\sin B = -4/5$ $\tan B = -4/3$

$\tan C = -60/11$ and $90° \le C \le 180°$

$\sin C = \frac{60}{61}$ $\cos C = \frac{-11}{61}$

to find the exact values of:

1. Sin (A-B) $= \sin A \cos B - \cos A \sin B$

$= \left(\frac{-8}{17}\right)\left(\frac{3}{5}\right) - \left(\frac{15}{17}\right)\left(\frac{-4}{5}\right)$

$= \frac{-24+60}{85} = \boxed{\frac{36}{85}}$

2. Cos 2A $= \cos^2 A - \sin^2 A$

$= \left(\frac{15}{17}\right)^2 - \left(\frac{-8}{17}\right)^2$

$= \frac{225-64}{289} = \boxed{\frac{161}{289}}$

3. Tan $\left(\frac{1}{2}C\right)$ $= \frac{\sin C}{1+\cos C} = \frac{60/61}{1+(-11/61)}$

$= \frac{60}{50} = \boxed{6/5}$

4. Csc (A+B) $= \frac{1}{\sin A \cos B + \cos A \sin B}$

$= \frac{1}{\left(\frac{-8}{17}\right)\left(\frac{3}{5}\right) + \left(\frac{15}{17}\right)\left(\frac{-4}{5}\right)}$

$= \frac{85}{-24-160} = \boxed{-\frac{85}{184}}$

5. Tan 2B $= \frac{2\tan B}{1-\tan^2 B}$

$= \frac{-4/3}{1-(-4/3)^2} = \frac{2(-4/3)}{1-\frac{16}{9}}$

$= \frac{-8/3}{-7/9} = + \boxed{\frac{24}{7}}$

6. Cos $\left(\frac{1}{2}A\right)$ $= -\sqrt{\frac{1}{2}(1+\cos A)}$

$= -\sqrt{\frac{1}{2}\left(1+\frac{15}{17}\right)}$

$= -\sqrt{\frac{1}{2}(32/17)} = \boxed{-4/\sqrt{17}}$

7. Prove: $\dfrac{2\csc^2 y-7\cot y-6}{6\csc^2 y-5\cot y-10}=\dfrac{\cot y-4}{3\cot y-4}$

$$\frac{2(\cot^2 y+1)-7\cot y-6}{6(\cot^2 y+1)-5\cot y-10}$$

$$\frac{2\cot^2 y-7\cot y-4}{6\cot^2 y-5\cot y-4}$$

$$\frac{\cancel{(2\cot y+1)}(\cot y-4)}{\cancel{(2\cot y+1)}(3\cot y-4)}$$

9. Solve exactly for A:

$\cos^2 A-\sin^2 A=\dfrac{\sqrt{3}}{2}$

$$\cos 2A=\frac{\sqrt{3}}{2}$$

$$2A=\pm\frac{\pi}{6}\pm 2\pi n$$

$$A=\pm\frac{\pi}{12}\pm\pi n$$

8. Prove: $\dfrac{\cot A\ \cot B-1}{\cot A\ +\cot B}=Cot(A+B)$

$$\frac{\frac{1}{\tan A}\cdot\frac{1}{\tan B}-1}{\frac{1}{\tan A}+\frac{1}{\tan B}}=\frac{1-\tan A\tan B}{\tan A+\tan B}$$

$$\frac{\frac{1}{\tan A\tan B}-\frac{\tan A\tan B}{\tan A\tan B}}{\frac{\tan B}{\tan A\tan B}+\frac{\tan A}{\tan A\tan B}}$$

$$\frac{1-\tan A\tan B}{\tan B+\tan A}$$

10. Solve ~~exactly~~ for x :

$\csc^2 x+\dfrac{\cos^2\frac{1}{2}x-\sin^2\frac{1}{2}x}{2\cos\frac{1}{2}x\ \sin\frac{1}{2}x}=3$

$$\csc^2 x+\frac{\cos 2(\frac{1}{2}x)}{\sin 2(\frac{1}{2}x)}=3$$

$$\cot^2 x+1+\cot x-3=0$$

$$\cot^2 x+\cot x-2=0$$

$$(\cot x+2)(\cot x-1)=0$$

$\cot x=-2$

$\tan x=-\frac{1}{2}$

$x=\tan^{-1}(-\frac{1}{2})$

$=-.464\pm\pi n$

$\cot x=1$

$\tan x=1$

$x=\frac{\pi}{4}\pm\pi n$

PreCalculus
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Page 2

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$\tan C = -60/11$ and $90° \le C \le 180°$

to find the exact values of:

1. cos (A-B) $= \cos A \cos B + \sin A \sin B$

$$= \frac{15}{17}\left(\frac{3}{5}\right) + \left(\frac{-8}{17}\right)\left(\frac{-4}{5}\right)$$

$$= \frac{45 + 32}{85}$$

$$= \frac{77}{85}$$

2. Sin 2B $= 2 \sin B \cos B$

$$= 2\left(\frac{-4}{5}\right)\left(\frac{3}{5}\right)$$

$$= -\frac{24}{25}$$

3. $\text{Sin}\left(\frac{1}{2}C\right) = +\sqrt{\frac{1}{2}\left(1-\left(\frac{-11}{61}\right)\right)}$

$$= \sqrt{\frac{1}{2}\left(\frac{72}{61}\right)} = \frac{6}{\sqrt{61}}$$

4. Tan (A+B) $= \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{\frac{-8}{15} + \frac{-4}{3}}{1 - \left(\frac{-8}{15}\right)\left(\frac{-4}{3}\right)} = \frac{\frac{-24-60}{45}}{1 - 32/45}$$

$$= \frac{-84}{12} = -7$$

5. Cot 2A $= \dfrac{1 - \tan^2 A}{2 \tan A}$

$$= \frac{1 - \left(-8/15\right)^2}{2\left(-8/15\right)}$$

$$= \frac{1 - \frac{64}{225}}{-16/15} = \frac{161}{\cancel{225}_{15}} \cdot \left(\frac{-\cancel{15}^{1}}{16}\right) = \frac{-161}{240}$$

6. $\text{Sec}\left(\frac{1}{2}B\right) = \frac{1}{+\sqrt{\frac{1}{2}\left(1 + 3/5\right)}}$

$$= \frac{1}{\sqrt{4/5}} = \sqrt{5}/2$$

7. Prove:

$$\sin\frac{4x}{7}\cos\frac{3x}{7}+\cos\frac{4x}{7}\sin\frac{3x}{7}=\sin x$$

$\sin\left(\frac{4x}{7}+\frac{3x}{7}\right)$

$=\sin\frac{7x}{7}$

$=\sin x$

9. Prove: $\cot A-\tan A=2\cot 2A$

$\frac{1}{\tan A}-\tan A=\cancel{2}\left(\frac{1-\tan^2 A}{\cancel{2}\tan A}\right.$

$\frac{1-\tan^2 A}{\tan A}$

10. Solve for x:

$$\frac{4}{\csc^2 x}+\frac{4}{\sec^2 x}=\tan^2 x$$

$4\sin^2 x+4\cos^2 x=\tan^2 x$

$\tan^2 x=4$

$\tan x=\pm 2$

$\tan x=2$

$x=\tan^{-1}2$

$=1.107\pm\pi n$

$\tan x=-2$

$x=\tan^{-1}(-2)$

$=-1.107\pm\pi n$

8. Solve for $x\in[0,\ \pi)$:

$$\tan\left(\frac{1}{2}x\right)+1=\cos x$$

$\frac{\sin x}{1+\cos x}=\cos x-1$

$\sin x=\cancel{\sin^2 x}\cos^2 x-1$

$\sin x=-\sin^2 x$

$\sin^2 x+\sin x=0$

$\sin x=-1$ or $\sin x=0$

$x=\frac{3\pi}{2}\pm 2\pi n$

$x=0\pm\pi n$

$x=0$

PreCalculus
Trig Identities Group Test v3
Page 3

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Score__________

For Problem 1-6, use:

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$\cos B = \frac{3}{5}$ and $-90° \le B \le 0°$; and

$\tan C = \frac{-60}{11}$ and $90° \le C \le 180°$

to find the exact values of:

1. Tan (A-B)

$$= \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{-8/15 - (-4/3)}{1 + (-8/15)(-4/3)}$$

$$= \frac{\frac{-24+60}{45}}{1 + \frac{32}{45}}$$

$$= \frac{\cancel{36}}{\cancel{77}} = 36/77$$

2. Cot 2B

$$= \frac{1 - \tan^2 B}{2 \tan B}$$

$$= \frac{1 - (-4/3)^2}{2(-4/3)} = \frac{1 - \frac{16}{9}}{-8/3}$$

$$= \frac{-7}{24}$$

3. $\csc\left(\frac{1}{2}C\right)$

$$= \frac{1}{\sqrt{\frac{1}{2}\left(1 - \left(-11/61\right)\right)}}$$

$$= \frac{\sqrt{61}}{6}$$

4. Cos (A+C) $= \cos A \cos C - \sin A \sin C$

$$\left(\frac{15}{17}\right)\left(\frac{3}{5}\right) - \left(\frac{-8}{17}\right)\left(\frac{-4}{5}\right)$$

$$= \frac{45 - 32}{85}$$

$$= \frac{13}{85}$$

5. Csc 2C

$$= \frac{1}{2 \sin C \cos C}$$

$$= \frac{1}{2\left(\frac{60}{61}\right)\left(\frac{-11}{61}\right)}$$

$$= -\frac{3721}{1320}$$

6. $\sin\left(\frac{1}{2}B\right)$

$$= -\sqrt{\frac{1}{2}(1 - \cos B)}$$

$$= -\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = -1/\sqrt{5}$$

7. Prove: $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$

$\frac{2\sin x \cos x}{\sin x} - \frac{2\cos^2 x - 1}{\cos x}$

$2\cos x - 2\cos x - \frac{1}{\cos x}$

$\frac{1}{\cos x}$

$\sec x$

9. Solve exactly for x :
$\tan^2 x - \sec^2 x - \sec x - \cancel{2}\,1 = 0$

$-1 - \sec x - 1 = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$x = \pm \frac{\pi}{3} \pm 2\pi n$

8. Prove: $\left(\sin\frac{1}{2}x + \cos\frac{1}{2}x\right)^2 = 1 + \sin x$

$\sin^2 \frac{1}{2}x + 2\sin\frac{1}{2}x\cos\frac{1}{2}x + \cos^2\frac{1}{2}x$

$\left(\sin^2\frac{1}{2}x + \cos^2\frac{1}{2}x\right) + 2\sin\frac{1}{2}x\cos\frac{1}{2}x$

$1 + \sin 2\left(\frac{1}{2}x\right)$

$1 + \sin x$

10. Solve for $\theta \in [0°, 360°)$
$\cos 3\theta \cos 12° = \sin 3\theta \sin 12°$

$\cos 3\theta \cos 12 - \sin 3\theta \sin 12° = 0$

$\cos(3\theta + 12°) = 0$

$3\theta + 12 = \pm 90 \pm 360n$

$3\theta = \begin{cases} 78 \pm 360n \\ -102 \pm 360n \end{cases}$

$\theta = \begin{cases} 26 \pm 120n \\ -34 \pm 120n \end{cases}$

$\theta \in \{26°, 146°, 266°, 86°, 206°, 326°\}$

PreCalculus
Trig Identities Group Test v3
Page 4

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Score__________

For Problem 1-6, use:

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$\cos B = \frac{3}{5}$ and $-90° \le B \le 0°$; and

$\tan C = \frac{-60}{11}$ and $90° \le C \le 180°$

to find the exact values of:

1. Csc (A-B) $= \dfrac{1}{\sin A \cos B - \cos A \sin B}$

$= \dfrac{1}{\left(\frac{-8}{17}\right)\left(\frac{3}{5}\right) - \left(\frac{15}{17}\right)\left(-\frac{4}{5}\right)}$

$= \dfrac{85}{-24 + 60} = \boxed{\dfrac{85}{36}}$

2. Sin 2A $= 2\sin A \cos A$

$= 2\left(\frac{-8}{17}\right)\left(\frac{15}{17}\right)$

$= \boxed{\dfrac{-240}{289}}$

3. $\text{Cos}\left(\frac{1}{2}C\right) = -\sqrt{\frac{1}{2}(1+\cos C)}$

$= \cancel{\pm}-\sqrt{\frac{1}{2}\left(1+\left(\frac{-11}{61}\right)\right)}$

$= \boxed{-5/\sqrt{61}}$

4. Sin (B+C) $= \sin B \cos C + \cos B \sin C$

$= \left(\frac{-4}{5}\right)\left(\frac{-11}{61}\right) + \left(\frac{3}{5}\right)\left(\frac{60}{61}\right)$

$= \dfrac{44 + 180}{305}$

$= \boxed{\dfrac{224}{305}}$

5. Cos 2C $= \cos^2 C - \sin^2 C$

$= \left(\frac{-11}{61}\right)^2 - \left(\frac{60}{61}\right)^2$

$= \dfrac{121 - 3600}{3721}$

$= \boxed{\dfrac{3479}{3721}}$

6. $\text{Cot}\left(\frac{1}{2}A\right) = \dfrac{1+\cos A}{\sin A}$

$= \dfrac{1 + 15/17}{-8/17} = \boxed{-4}$

7. Prove: $\tan\frac{1}{2}x+\cot\frac{1}{2}x=\csc\frac{1}{2}x\sec\frac{1}{2}x$

$u = 1/2 x$

$x = 2u$

$\tan u + \cot u = \csc 2u \sec 2u$

$$\frac{\sin u}{\cos u} + \frac{\cos u}{\sin u} = \frac{1}{\sin u}\ \frac{1}{\cos u}$$

$$\frac{\sin^2 u + \cos^2 u}{\cos u \sin u}$$

$$\frac{1}{\cos u \sin u} = \frac{1}{\sin u \cos u}$$

9. Prove: $\frac{\sec\beta}{\sin\beta}-\frac{\sin\beta}{\cos\beta}=\cot\beta$

$$\frac{1-\sin^2\beta}{\sin\beta\cos\beta}$$

$$\frac{\cos^2\beta}{\sin\beta\cos\beta}$$

$$\frac{\cos\beta}{\sin\beta}$$

$\cot\beta$

8. Solve for $\mu\in(-90°,\ 270°)$

$\sin\mu\ \sin 60° - \cos\mu\ \cos 60° = \frac{1}{2}$

$\cos(\mu+60°) = -1/2$

$\mu + 60° = \cos^{-1}(-1/2)$

$\mu + 60° = \pm 120 \pm 360n$

$$\mu = \begin{cases} 60 \pm 360n \\ -180 \pm 360n \end{cases}$$

$\mu \in \{60°, 180°\}$

10. Solve for $x\in(-\pi,\pi)$: $\frac{2}{\cot x\tan 2x}=-2$

$$\frac{\cancel{2}}{\cancel{\cot x}\left(\frac{\cancel{2\tan x}}{1-\tan^2 x}\right)} = -2$$

$1-\tan^2 x = -2$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \pm\frac{\pi}{3} \pm \pi n$